# Active thermography based on tensor rank decomposition

Thermosense: Thermal Infrared Applications XLIV

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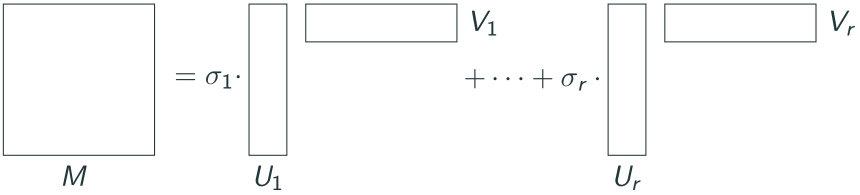
# The Singular Value Decomposition (SVD) of a matrix

The SVD of a real $m \times n$ matrix $M \in \mathbb{R}^{m \times n}$ of rank-$r$ is

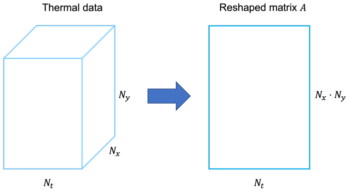
$$M = U\Sigma V^T = \sum_{k=1}^{r} \sigma_k U_k \cdot V_k, \quad \sigma_1 \geq \sigma_2 \geq \cdots \geq \sigma_r \geq 0.$$




The Eckart-Young theorem says that the truncation $M_k = U\Sigma_k V^T$ is the best rank-$k$ approximation to $M$.

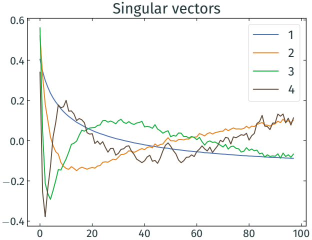
# SVD Thermography

1) Thermal data is reshaped into a matrix $A$

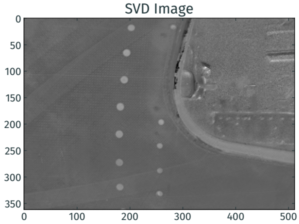



that is normalized along its columns.

2) Right singular vectors of $A$ are computed:



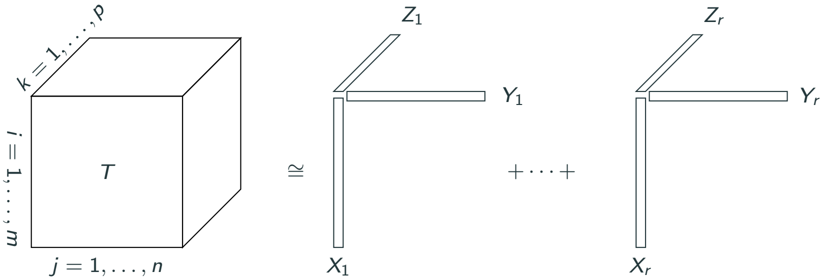

3) Features $\phi$ computed along rows by averaging, e.g., with respect to second singular vector:




Rajic, N., *Composite Structures* (2002).

# The CANDECOMP-PARAFAC (CP) Decomposition of a tensor

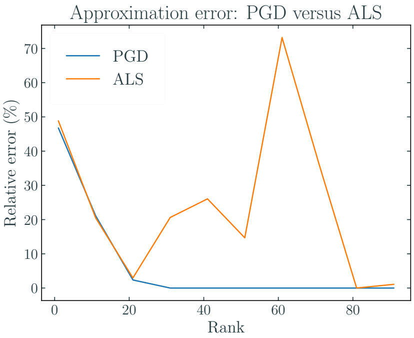
The CP Decomposition of a 3rd-order tensor $T \in \mathbb{R}^{m \times n \times p}$ is an approximation $S$ of the form $T \cong S = \sum_{l=1}^{r} x_l \cdot y_l \cdot z_l$.

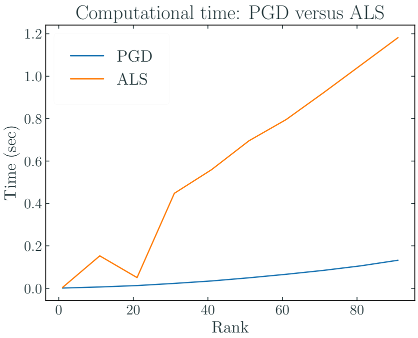



Although there is no analog to the Eckart-Young theorem in this setting, useful approximations can be found by attempting to minimize the objective $S \mapsto \| T - S \|_F$.

# Computing the CP Decomposition: PGD vs ALS

Performance on random tensor of size $25 \times 25 \times 25$. ALS computed using TensorLy[1].



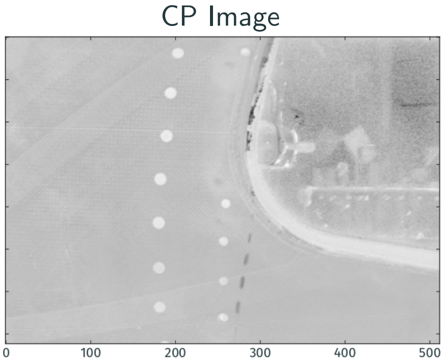




[1] Kossaifi, J., et al., *J Machine Learning Res* (2019).

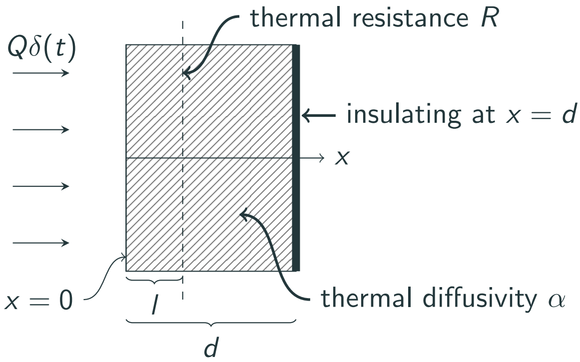
PGD: Proper Generalized Decomposition; ALS: Alternating Least Squares

# SVD versus CP Thermography

CP Decomposition avoids flattening the spatial dimensions.

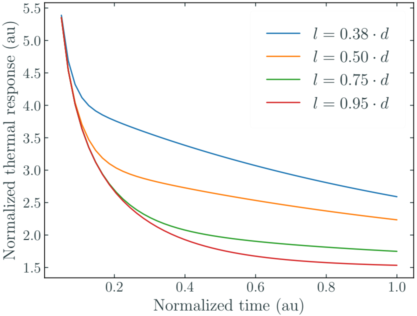



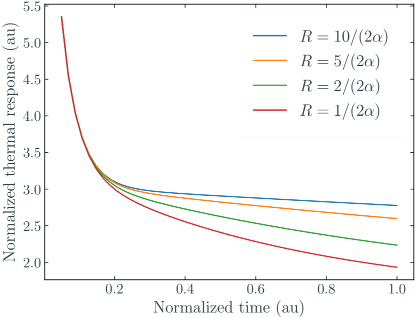
# One-dimensional thermal model $\Theta(x, t; l, R)$




- The surface temperature can be expressed analytically in the Laplace domain.
- In the model, the parameters $d$ and $\alpha$ are fixed, and $l$ and $R$ are allowed to vary.
- No radiation or convective losses in the model.

# Model output $\Theta(x = 0, t; l, R)$




Surface temperature in the time domain computed numerically by inverting the Laplace transform using de Hoog's algorithm.

# Fixed CP Decomposition thermography

1) A 3rd-order tensor $T \in \mathbb{R}^{m \times n \times p}$ is generated from thermal model $\Theta$:

$$T_{ijk} = \Theta(0, t_k; l_i, R_j).$$

2) Its rank-$r$ CP Decomposition computed using PGD:

$$S = \sum_{l=1}^{r} \lambda_l X_l \cdot Y_l \cdot Z_l.$$

$Z_l$ represents time dimension of data.

3) A feature vector $\vec{\phi}$ associated to any temperature profile $u$ is given by

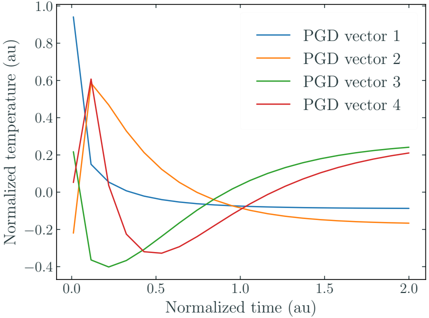
$$\vec{\phi} = Z^T \cdot u \in \mathbb{R}^r,$$

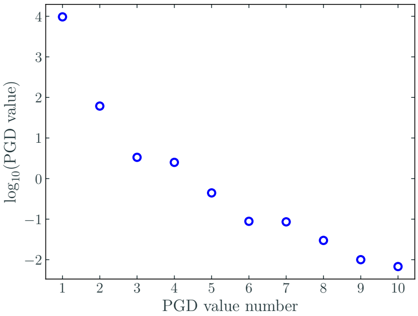
where $Z$ is the matrix with columns $Z_1, \ldots, Z_r$.

Based on Cramer, K. E. and Winfree, W. P., *Thermosense XXXIII* (2011).

# Time components of the CP decomposition of $T$

Example CP vectors for $T \in \mathbb{R}^{4\times7\times20}$.

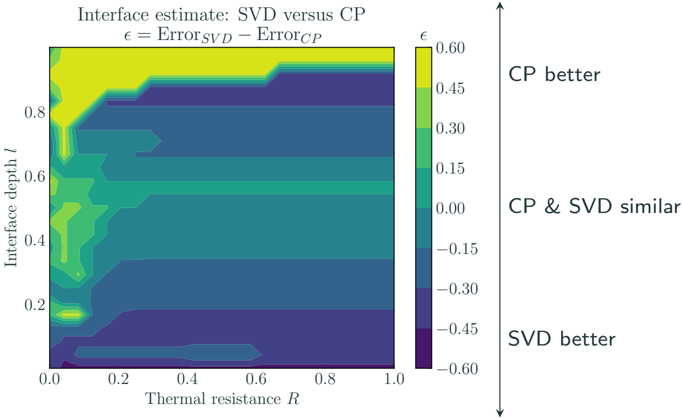



Note that the components of the CP Decomposition are not necessarily orthogonal.

# Simulation & Results

1. Compute feature vectors $\vec{\phi}_{SVD}$ and $\vec{\phi}_{CP}$ for each of the 28 combinations of depth and thermal resistance.
2. Compute feature vectors for each combination $(l, R) \in (0, d) \times (0, 1)$.
3. Depth and resistance estimates $\hat{R}$ and $\hat{l}$ for each $(l, R)$ pair in Step 2 defined by nearest feature vector from Step 1.
4. Compare errors using both methods.



# Conclusions

- The PGD method provides an efficient method for computing CP Decompositions and compares well against ALS in terms of accuracy and efficiency.
- Tests on simulated data show that CP Thermography can in some circumstances perform as well or better than Fixed Eigenvector Thermography. Comparing both methods on experimental data with known defects is the logical next step.
- Questions?

Thank you!